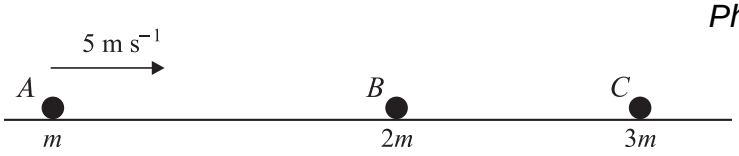
OCR Maths M2

Topic Questions from Papers

Collisions

1



Three smooth spheres A, B and C, of equal radius and of masses m kg, $2m$ kg and $3m$ kg respectively, lie in a straight line and are free to move on a smooth horizontal table. Sphere A is moving with speed $5\,\text{m s}^{-1}$ when it collides directly with sphere B which is stationary. As a result of the collision B starts to move with speed $2\,\text{m s}^{-1}$.

(i) Find the coefficient of restitution between A and B. [4]

(ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B subsequently collides with sphere C which is stationary. As a result of this impact B and C coalesce.

(iii) Show that there will be another collision. [3]

(Q4, June 2005)

2 Three smooth spheres A, B and C, of equal radius and of masses $3m$ kg, $2m$ kg and m kg respectively, are free to move in a straight line on a smooth horizontal table. Spheres B and C are stationary. Sphere A is moving with speed $2\,\text{m s}^{-1}$ when it collides directly with sphere B. The collision is perfectly elastic.

(i) Find the velocities of A and B after the collision. [6]

(ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B continues its motion and hits C. After the collision, B continues in the same direction with speed $1.0\,\text{m s}^{-1}$ and C moves with speed $2.8\,\text{m s}^{-1}$.

(iii) Find the coefficient of restitution between B and C. [2]

(Q5, Jan 2006)

3 A small sphere of mass 0.3 kg is dropped from rest at a height of 2 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.4 m above the ground. Ignoring air resistance, calculate the magnitude of the impulse which the ground exerts on the sphere when it rebounds. [5]

(Q2, June 2006)

4 Two uniform smooth spheres, A and B, have the same radius. The mass of A is 2 kg and the mass of B is m kg. Sphere A is travelling in a straight line on a smooth horizontal surface, with speed $5\,\text{m s}^{-1}$, when it collides directly with sphere B, which is at rest. As a result of the collision, sphere A continues in the same direction with a speed of $2\,\text{m s}^{-1}$.

(i) Find the greatest possible value of m. [3]

It is given that $m = 1$.

(ii) Find the coefficient of restitution between A and B. [3]

On another occasion A and B are travelling towards each other, each with speed $5\,\text{m s}^{-1}$, when they collide directly.

(iii) Find the kinetic energy lost due to the collision. *(Q8, June 2006)* [8]

5 Two smooth spheres A and B, of equal radius and of masses 0.2 kg and 0.1 kg respectively, are free to move on a smooth horizontal table. A is moving with speed $4\,\text{m s}^{-1}$ when it collides directly with B, which is stationary. The collision is perfectly elastic. Calculate the speed of A after the impact. [4]

(Q2, Jan 2007)

6 A small sphere of mass 0.2 kg is projected vertically downwards with speed $21\,\text{m s}^{-1}$ from a point at a height of 40 m above horizontal ground. It hits the ground and rebounds vertically upwards, coming to instantaneous rest at its initial point of projection. Ignoring air resistance, calculate

(i) the coefficient of restitution between the sphere and the ground, [6]

(ii) the magnitude of the impulse which the ground exerts on the sphere. [2]

(Q3, Jan 2007)

7 Two small spheres A and B, with masses 0.3 kg and m kg respectively, lie at rest on a smooth horizontal surface. A is projected directly towards B with speed $6\,\text{m s}^{-1}$ and hits B. The direction of motion of A is reversed in the collision. The speeds of A and B after the collision are $1\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively. The coefficient of restitution between A and B is e.

(i) Show that $m = 0.7$. [2]

(ii) Find e. [2]

B continues to move at $3\,\text{m s}^{-1}$ and strikes a vertical wall at right angles. The coefficient of restitution between B and the wall is f.

(iii) Find the range of values of f for which there will be a second collision between A and B. [2]

(iv) Find, in terms of f, the magnitude of the impulse that the wall exerts on B. [3]

(v) Given that $f = \frac{3}{4}$, calculate the final speeds of A and B, correct to 1 decimal place. [7]

(Q7, June 2007)

8 A ball is projected with speed $12\,\text{m s}^{-1}$ at an angle of elevation of 55° above the horizontal. At the instant when the ball reaches its greatest height, it hits a vertical wall, which is perpendicular to the ball's path. The coefficient of restitution between the ball and the wall is 0.65. Calculate the speed of the ball

(i) immediately before its impact with the wall, [2]

(ii) immediately after its impact with the wall. [2]

(Q1, Jan 2008)

9 A particle P of mass $2m$ is moving on a smooth horizontal surface with speed u when it collides directly with a particle Q of mass km whose speed is $3u$ in the opposite direction. As a result of the collision, the directions of motion of both particles are reversed and the speed of P is halved.

(i) Find, in terms of u and k, the speed of Q after the collision. Hence write down the range of possible values of k. [4]

(ii) Calculate the magnitude of the impulse which Q exerts on P. [2]

(iii) Given that $k = \frac{1}{2}$, calculate the coefficient of restitution between P and Q. [3]

(Q5, Jan 2008)

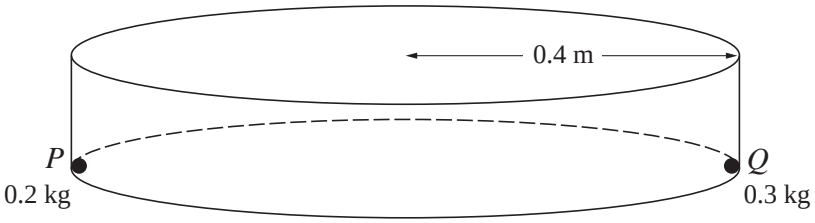
10

Two small spheres A and B of masses 2 kg and 3 kg respectively lie at rest on a smooth horizontal platform which is fixed at a height of 4 m above horizontal ground (see diagram). Sphere A is given an impulse of 6 N s towards B, and A then strikes B directly. The coefficient of restitution between A and B is $\frac{2}{3}$.

(i) Show that the speed of B after it has been hit by A is $2\,\text{m s}^{-1}$. [6]

(Q7, June 2008)

11



Two small spheres, P and Q, are free to move on the inside of a smooth hollow cylinder, in such a way that they remain in contact with both the curved surface and the base of the cylinder. The mass of P is 0.2 kg, the mass of Q is 0.3 kg and the radius of the cylinder is 0.4 m. P and Q are stationary at opposite ends of a diameter of the base of the cylinder (see diagram). The coefficient of restitution between P and Q is 0.5. P is given an impulse of magnitude 0.8 N s in a tangential direction.

(i) Calculate the speeds of the particles after P's first impact with Q. **[8]**

Q subsequently catches up with P and there is a second impact.

(ii) Calculate the speeds of the particles after this second impact. **[7]**

(iii) Calculate the magnitude of the force exerted on Q by the curved surface of the cylinder after the second impact. **[2]**

(Q7, Jan 2009)

12 Two uniform spheres, A and B, have the same radius. The mass of A is 0.4 kg and the mass of B is 0.2 kg. The spheres A and B are travelling in the same direction in a straight line on a smooth horizontal surface, A with speed $5\,\text{m s}^{-1}$, and B with speed $v\,\text{m s}^{-1}$, where $v < 5$. A collides directly with B and the impulse between them has magnitude 0.9 N s. Immediately after the collision, the speed of B is $6\,\text{m s}^{-1}$.

(i) Calculate v. **[3]**

B subsequently collides directly with a stationary sphere C of mass 0.1 kg and the same radius as A and B. The coefficient of restitution between B and C is 0.6.

(ii) Determine whether there will be a further collision between A and B. **[10]**

(Q6, June 2009)

13 A small sphere of mass 0.2 kg is dropped from rest at a height of 3 m above horizontal ground. It falls vertically, hits the ground and rebounds vertically upwards, coming to instantaneous rest at a height of 1.8 m above the ground.

(i) Calculate the magnitude of the impulse which the ground exerts on the sphere. **[5]**

(ii) Calculate the coefficient of restitution between the sphere and the ground. **[2]**

(Q2, Jan 2010)

14 Two spheres of the same radius with masses 2 kg and 3 kg are moving directly towards each other on a smooth horizontal plane with speeds $8\,\text{m s}^{-1}$ and $4\,\text{m s}^{-1}$ respectively. The spheres collide and the kinetic energy lost is 81 J. Calculate the speed and direction of motion of each sphere after the collision. **[12]**

(Q5, Jan 2010)

15 A particle A of mass $2m$ is moving with speed u on a smooth horizontal surface when it collides with a stationary particle B of mass m. After the collision the speed of A is v, the speed of B is $3v$ and the particles move in the same direction.

(i) Find v in terms of u. **[3]**

(ii) Show that the coefficient of restitution between A and B is $\frac{4}{5}$. **[2]**

B subsequently hits a vertical wall which is perpendicular to the direction of motion. As a result of the impact, B loses $\frac{3}{4}$ of its kinetic energy.

(iii) Show that the speed of B after hitting the wall is $\frac{3}{5}u$. **[4]**

(iv) B then hits A. Calculate the speeds of A and B, in terms of u, after this collision and state their directions of motion. **[8]**

(Q6, June 2010)

16 Three small smooth spheres A, B and C of masses 0.2 kg, 0.7 kg and m kg respectively are free to move in a straight line on a smooth horizontal table. Initially B and C are stationary and A is moving with velocity $1.8\,\text{m s}^{-1}$ directly towards B. The coefficient of restitution for the collision between A and B is e. Immediately after this collision the speed of A is greater than the speed of B.

(i) Calculate the set of possible values of e. **[9]**

It is now given that the speed of B immediately after the collision with A is $0.75\,\text{m s}^{-1}$. B continues its motion and strikes C directly in a perfectly elastic collision. B has speed $0.25\,\text{m s}^{-1}$ immediately after its collision with C.

(ii) Calculate the two possible values of m. **[6]**

(Q7, Jan 2011)

17 Two small spheres A and B are moving towards each other along a straight line on a smooth horizontal surface. A has speed $3\,\text{m s}^{-1}$ and B has speed $1.5\,\text{m s}^{-1}$ before they collide directly. The direction of motion of B is reversed in the collision. The speeds of A and B after the collision are $2\,\text{m s}^{-1}$ and $2.9\,\text{m s}^{-1}$ respectively.

(i) (a) Show that the direction of motion of A is unchanged by the collision. **[2]**

(b) Calculate the coefficient of restitution between A and B. **[2]**

The mass of B is 0.2 kg.

(ii) Find the mass of A. **[3]**

B continues to move at $2.9\,\text{m s}^{-1}$ and strikes a vertical wall at right angles. The wall exerts an impulse of magnitude 0.68 N s on B.

(iii) Calculate the coefficient of restitution between B and the wall. **[4]**

(Q4, June 2011)

18 A small ball of mass 0.5 kg is held at a height of 3.136 m above a horizontal floor. The ball is released from rest and rebounds from the floor. The coefficient of restitution between the ball and floor is e.

(i) Find in terms of e the speed of the ball immediately after the impact with the floor and the impulse that the floor exerts on the ball. **[4]**

The ball continues to bounce until it eventually comes to rest.

(ii) Show that the time between the first bounce and the second bounce is $1.6e$. **[2]**

(iii) Write down, in terms of e, the time between

(a) the second bounce and the third bounce,

(b) the third bounce and the fourth bounce. **[2]**

(iv) Given that the time from the ball being released until it comes to rest is 5 s, find the value of e. **[5]**

(Q6, Jan 2012)

19 A particle, of mass 0.8 kg, moves along a smooth horizontal surface. It hits a vertical wall, which is at right angles to the direction of motion of the particle, and rebounds. The speed of the particle as it hits the wall is $4\,\text{m s}^{-1}$ and the coefficient of restitution between the particle and the wall is 0.3. Find

(i) the impulse that the wall exerts on the particle, **[3]**

(ii) the kinetic energy lost in the impact. **[2]**

(Q1, June 2012)

20 Three particles A, B and C are in a straight line on a smooth horizontal surface. The particles have masses 0.2 kg, 0.4 kg and 0.6 kg respectively. B is at rest. A is projected towards B with a speed of $1.8\,\text{m s}^{-1}$ and collides with B. The coefficient of restitution between A and B is $\frac{1}{3}$.

(i) Show that the speed of B after the collision is $0.8\,\text{m s}^{-1}$ and find the speed of A after the collision. **[6]**

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C is moving with speed $0.2\,\text{m s}^{-1}$ in the same direction as B. Particle B subsequently collides with C. The coefficient of restitution between B and C is e.

(ii) Find the set of values for e such that B does not collide again with A. **[7]**

(Q6, June 2012)

21 A particle A is released from rest from the top of a smooth plane, which makes an angle of 30° with the horizontal. The particle A collides 2 s later with a particle B, which is moving up a line of greatest slope of the plane. The coefficient of restitution between the particles is 0.4 and the speed of B immediately before the collision is $2\,\text{m s}^{-1}$. B has velocity $1\,\text{m s}^{-1}$ down the plane immediately after the collision. Find

(i) the speed of A immediately after the collision, **[4]**

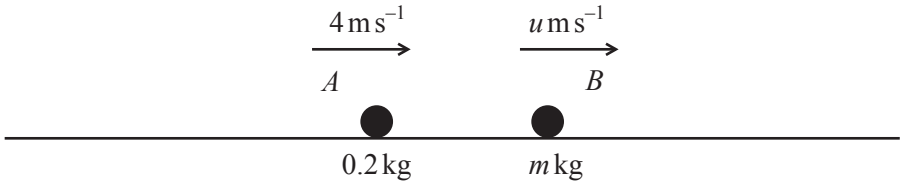
(ii) the distance A moves up the plane after the collision. **[2]**

The masses of A and B are 0.5 kg and m kg, respectively.

(iii) Find the value of m. **[3]**

(Q3, Jan 2013)

22



The masses of two particles A and B are 0.2 kg and m kg respectively. The particles are moving with constant speeds $4\,\text{m s}^{-1}$ and $u\,\text{m s}^{-1}$ in the same horizontal line and in the same direction (see diagram). The two particles collide and the coefficient of restitution between the particles is e. After the collision, A and B continue in the same direction with speeds $4(1 - e + e^2)\,\text{m s}^{-1}$ and $4\,\text{m s}^{-1}$ respectively.

(i) Find u and m in terms of e. **[6]**

(ii) Find the value of e for which the speed of A after the collision is least and find, in this case, the total loss in kinetic energy due to the collision. **[5]**

(iii) Find the possible values of e for which the magnitude of the impulse that B exerts on A is 0.192 N s. **[4]**

(Q6, June 2013)